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Finite-time H_∞ control for linear systems with semi-Markovian switching

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Abstract Finite-time H_∞ control problem is a fascinating and hot issue in the field of control science. This paper presents a novel framework for finite-time H_∞ stabilization of semi-Markovian switching system. By employing Lyapunov–Krasovskii functional and matrix inequality techniques, together with properties of semi-Markovian process, sufficient conditions are proposed to guarantee finite-time boundedness, H_∞ finite-time boundedness and finite-time H_∞ state feedback stabilization for semi-Markovian switching system. At the same time, a state feedback controller is provided to ensure that the proposed closed-loop system is finite-time H_∞ stabilization. Finally, a numerical example and simulations are given to show the correctness and effectiveness of the proposed results.

Keywords Semi-Markovian switching system · Finite-time bounded · Finite-time H_∞ stabilization · Feedback controller

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1 Introduction

It is well known that Markovian switching systems have come to play an increasingly important role in many areas of applications including electric power systems [1], communication systems [2], manufacturing systems [3], biological systems [4] and economic systems [5]. Over the past few decades, considerable attention has been devoted to the analysis of Markovian switching systems and a large number of results are developed, see [6–17], and the references therein. However, in many practical applications, if a real system does not satisfy the so-called memoryless restriction, the widely used Markovian switching scheme would not be applicable. In other words, Markovian switching systems have certain limitations in some senses since the jump time of a Markovian process obeys exponential distribution, while the sojourn-time of semi-Markovian switching systems can extend to more general distribution. In this paper, we consider the sojourn-time follows Weibull distribution when the semi-Markovian switching system jumps from mode i to mode j, and attempt to establish finite-time H_∞ stabilization condition for the semi-Markovian switching system.

Actually, most of the modeling, analysis and design results for Markovian switching systems should be regarded as special cases of semi-Markovian switching systems. The reason is that transition rates are time-varying in semi-Markovian switching systems instead of constants in Markovian switching systems. Due to the relaxed conditions on the probability distributions, semi-Markovian switching systems have much more extensive application prospect than conventional Markovian switching systems, which can be found in [18–20] for more details. But, in contrast to substantial results for Markovian switching systems, little attention has been paid to developing numerically testable stability conditions and little research has been devoted to the feedback controller design concerning semi-Markovian switching systems, except for [21–29] where stochastic stabillity [21,22,24,28], H_∞ state feedback control [23], fault-tolerant control [25,26], sliding mode control [27], quantized control design [29]. Hence, it is of both theoretical merit and practical interest to investigate the stability, stabilization, filtering and control problem for semi-Markovian switching systems.

Since classic Lyapunov stability theory is over an infinite time interval and it could not reflect transient state performance, many research efforts have been devoted to the study of finite-time stability or short-time stability problem over the past few years [30–35]. To name a few, the output feedback finite-time stabilization for continuous linear system is investigated in [33]. The problem of finite-time stability and stabilization of linear stochastic systems is considered in [34]. With delays appearing in systems, the problem of finite-time boundedness for Markovian switching systems with disturbances is studied in [35]. On another research front line, in order to reject the instability of systems caused by the existence of external disturbances, the state feedback H_∞ control problem has been extensively studied. For instance, robust H_∞ sliding mode control for Markovian switching system is discussed in [36], robust non-fragile H_∞ control for switched neutral systems is investigated in [37], other useful and interesting results can be found in the literature [38–41]. In recent years, to guarantee finite-time bounded of the resulting system and reduce the effect of the disturbance input on the controlled output to a prescribed level, finite-time H_∞ control problem has been extensively studied in [42–44]. In [42], a finite-time H_∞ state feedback controller is designed for linear continuous systems. In [43], a finite-time H_∞ state feedback controller is provided to linear Markovian switching systems; In [44], a neural-network-based finite-time H_∞ controller is presented for a class of nonlinear Markovian switching systems. However, the finite-time H_∞ control scheme for Markovian switching systems will not be applicable in some situations, when the practical systems do not satisfy the so-called memoryless restriction and the mode change of the plant should be modeled as a semi-Markov process. To date and to the best of our knowledge, the problems of finite-time H_∞ control of semi-Markovian switching systems have not been investigated yet, which motivates our paper.

In this paper, we concentrate on the finite-time H_∞ control problem for semi-Markovian switching systems. Based on Lyapunov–Krasovskii functional and matrix inequality techniques, together with properties of semi-Markovian process, sufficient conditions and feedback controller are proposed to ensure finite-time H_∞ stabilization for the underlying semi-Markovian switching system. Finally, a numerical example is given to illustrate the effectiveness of the proposed methodology. The main contributions of this paper are summarized as follows: (i) Different from the existing references [21–29], a novel finite-time bounded condition is established for the semi-Markovian switching system; (ii) Furthermore, a novel H_∞ finite-time bounded condition is presented for the semi-Markovian switching system with external disturbances; (iii) Meanwhile, a finite-time H_∞ state feedback controller is provided to ensure finite-time H_∞ stabilization of semi-Markovian switching system with external disturbances.

The remainder of this paper is organized as follows: Sect. 2 contains the problem statement and preliminaries; Sect. 3 presents finite-time boundedness and finite-time H_∞ performance for semi-Markovian switching system; Sect. 4 provides a numerical example to verify the effectiveness of the proposed results; Concluding remarks are given in Sect. 5.

1.1 Notations

The following notations are used throughout the paper. $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ matrices. $X < Y (X > Y)$, where X and Y are both symmetric matrices, means that

$X - Y$ is negative(positive) definite. I is the identity matrix with proper dimensions. For a symmetric block matrix, we use $\star$ to denote the terms introduced by symmetry. $\mathscr{E}$ stands for the mathematical expectation. $\Gamma V(x(t), r(t), t)$ denotes the infinitesimal generator of $V(x(t), r(t), t)$. $\|v\|$ is the Euclidean norm of vector v, $\|v\| = (v^T v)^{\frac{1}{2}}$, while $\|A\|$ is spectral norm of matrix A, $\|A\| = [\lambda_{\max}(A^T A)]^{\frac{1}{2}}$. $\lambda_{\max(\min)}(A)$ is the eigenvalue of matrix A with maximum(minimum) real part. $L_2^n[0, +\infty)$ is the space of n-dimensional square integrable function vector over $[0, +\infty)$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2 problem statement and preliminaries

Given a complete probability space $\{\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathbf{P}\}$ with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is increasing and right continuous while $\mathscr{F}_0$ contains all P-null sets), where Ω is the sample space, $\mathscr{F}$ is the algebra of events and $\mathbf{P}$ is the probability measure defined on $\mathscr{F}$. Let $\{r(t), t \geq 0\}$ be a continuous-time semi-Markovian process taking values in a finite state space $S = \{1, 2, 3, \cdots, N\}$. The evolution of the semi-Markovian process $r(t)$ is governed by the following probability transitions:

$$P(r(t+h) = j | r(t) = i) = \begin{cases} \pi_{ij}(h)h + o(h) & i \neq j \\ 1 + \pi_{ii}(h)h + o(h) & i = j \end{cases}$$

where $h > 0$, $\lim_{h\to 0} \frac{o(h)}{h} = 0$, $\pi_{ij}(h) \geq 0 (i, j \in S, i \neq j)$ is the transition rate from mode i to j and for any state or mode, it satisfies

$$\pi_{ii}(h) = -\sum_{j=1, j\neq i}^{N} \pi_{ij}(h)$$

Remark 2.1 It should be pointed out that the probability distribution of sojourn-time has extended from exponential distribution to Weibull distribution, and the transition rate in semi-Markovian switching will be time-varying instead of constant in Markovian switching [22]. In practice, the transition rate $\pi_{ij}(h)$ is general bounded by $\underline{\pi}_{ij} \leq \pi_{ij}(h) \leq \overline{\pi}_{ij}$, $\underline{\pi}_{ij}$ and $\overline{\pi}_{ij}$ are real constant scalars. Then $\pi_{ij}(h)$ can always be described by $\pi_{ij}(h) = \pi_{ij} + \Delta\pi_{ij}$, where $\pi_{ij} = \frac{1}{2}(\underline{\pi}_{ij} + \overline{\pi}_{ij})$ and $|\Delta\pi_{ij}| \leq \lambda_{ij}$ with $\lambda_{ij} = \frac{1}{2}(\overline{\pi}_{ij} - \underline{\pi}_{ij})$.

The following linear system with semi-Markovian switching (semi-Markovian switching system) over the space $\{\Omega, \mathscr{F}, \mathbf{P}\}$ is investigated in this paper

$$\begin{aligned} \dot{x}(t) &= A(r(t))x(t) + B(r(t))u(t) + G(r(t))\delta(t) \\ y(t) &= C(r(t))x(t) + D(r(t))u(t) + H(r(t))\delta(t) \\ x(t_0) &= x_0, \quad r_{t_0} = r_0, \quad t_0 = 0 \end{aligned} \tag{1}$$

where $r(t)$ is a semi-Markovian process, $x(t) \in \mathbb{R}^n$ represents the state vector, $u(t)$ denotes the control input. $\delta(t) \in L_2^n[0, +\infty)$ is an arbitrary external disturbance. $y(t) \in \mathbb{R}^l$ is the control output; x_0, r_0, t_0 represent the initial state, initial mode, initial time, respectively. $A(r(t))$, $B(r(t))$, $C(r(t))$, $D(r(t))$, $G(r(t))$ and $H(r(t))$ are known mode-dependent constant matrices with appropriate dimensions.

In this paper, we consider the following state feedback controller:

$$u(t) = K(r(t))x(t) \tag{2}$$

where $K(r(t))$ is the state feedback gain to be designed.

For notational simplicity, we denote $A(r(t))$, $B(r(t))$, $C(r(t))$, $D(r(t))$, $G(r(t))$, $H(r(t))$, $K(r(t))$ by A_i, B_i, C_i, D_i, G_i, H_i, K_i for $r(t) = i \in S$. Then the closed-loop system can be written as follows:

$$\begin{aligned} \dot{x}(t) &= (A_i + B_i K_i)x(t) + G_i\delta(t) \\ y(t) &= (C_i + D_i K_i)x(t) + H_i\delta(t) \\ x(t_0) &= x_0, \quad r_{t_0} = r_0, \quad t_0 = 0 \end{aligned} \tag{3}$$

The purpose of this paper is to design the state feedback gain $K(r(t))$ for linear systems with semi-Markovian switching to achieve finite-time H_∞ state feedback stabilization. Before proceeding with the main results, we present the following assumptions, definitions and lemmas, which play an important role in the proof of the main result.

Assumption 2.1 For any given positive number τ and the actual working time T, the external disturbances input $\delta(t)$ is time-varying and satisfies the constraint condition

$$\int_{t_0}^{T} \delta^T(t)\delta(t)\mathrm{d}t \leq \tau, \qquad \tau \geq 0$$

Definition 2.1 Under the circumstance of $u(t) = 0$, for given positive constants c_1, τ, T and symmetric matrices $M_i > 0, i \in S$, the semi-Markovian switching system (1) is said to be finite-time bounded (FTB) with respect to (c_1, c_2, T, M_i, τ), if there exists a constant $c_2 > c_1$, such that

$$\mathscr{E}\{x_0^T M_i x_0\} \leq c_1 \Longrightarrow \mathscr{E}\{x^T(t) M_i x(t)\} \leq c_2, \quad \forall t \in [0, T]$$

Remark 2.2 It should be mentioned that the concept of finite-time bounded (FTB) will reduce to finite-time stable (FTS) [45,46] if the semi-Markovian switching system (1) does not exist the exogenous disturbances input, *i.e.*, $\delta(t) = 0$. That is to say, finite-time stability can be regarded as a particular case of finite-time boundedness under the circumstance of $u(t) = 0$.

Definition 2.2 Under the circumstance of $u(t) = 0$, the semi-Markovian switching system (1) is said to be H_∞ finite-time bounded with respect to (c_1, c_2, T, M_i, τ), if there exists a positive constant γ such that i) semi-Markovian switching system (1) is finite-time bounded with respect to (c_1, c_2, T, M_i, τ); ii) for all admissible $\delta(t)$ satisfying Assumption 2.1 and under zero initial condition $x_{t_0} = 0, t_0 = 0$, the system output $y(t)$ satisfies the following cost function inequality

$$\mathscr{E}\left\{\int_0^T y^T(t) y(t) \mathrm{d}t\right\} \leq \gamma^2 \int_0^T \delta^T(t)\delta(t)\mathrm{d}t$$

Definition 2.3 The semi-Markovian switching system (1) is said to be finite-time H_∞ state feedback stabilizable with respect to (c_1, c_2, T, M_i, τ), if there exist a positive constant γ and state feedback controller in form (2) such that the closed-loop system (3) is H_∞ finite-time bounded.

Lemma 2.1 [47] *Let τ_1 and τ_2 be bounded stopping times such that $0 \leq \tau_1 \leq \tau_2$,* a.s. *If $V(x(t), r(t), t)$ and $\Gamma V(x(t), r(t), t)$ are bounded on $t \in [\tau_1, \tau_2]$ with probability 1, then the following equality is held*

$$\mathscr{E}\{V(x(\tau_2), r(\tau_2), \tau_2)\} = \mathscr{E}\{V(x(\tau_1), r(\tau_1), \tau_1)\} + \mathscr{E}\int_{\tau_1}^{\tau_2} \Gamma V(x(s), r(s), s)\mathrm{d}s$$

Lemma 2.2 *Let $W \in \mathbb{R}^{n\times n}$ be a symmetric matrix, and let $x \in \mathbb{R}^n$, then the following inequality holds*

$$\lambda_{min}(W)x^T x \leq x^T W x \leq \lambda_{max}(W) x^T x$$

Lemma 2.3 [48] *Given any scalar ε and matrix $U \in \mathbb{R}^{n\times n}$, the following inequality*

$$\varepsilon(U + U^T) \leq \varepsilon^2 V + U V^{-1} U^T$$

holds for any symmetric positive definite matrix $V \in \mathbb{R}^{n\times n}$.

3 Main results

In this section, we will investigate the problem of finite-time H_∞ controller design for semi-Markovian switching system (1). To this end, firstly, the finite-time bounded condition is obtained by stochastic Lyapunov–Krasovskii functional technique and matrix inequality technique. Then some sufficient conditions will be provided ensuring the H_∞ finite-time boundedness and finite-time H_∞ state feedback stabilization for semi-Markovian switching system (1).

3.1 Finite-time bounded condition for semi-Markovian switching system

Theorem 3.1 *Under the circumstance of $u(t) = 0$, given $c_1 > 0$, $T > 0$, $M_i > 0, i \in S$, $\tau > 0$, semi-Markovian switching system (1) is said to be finite-time bounded with respect to (c_1, c_2, T, M_i, τ), if there exist positive constants $\alpha > 0$, $\rho > 0$, symmetric positive definite matrices $P_i \in \mathbb{R}^{n\times n}$, $i \in S$, symmetric positive definite matrices $V_{ij} \in \mathbb{R}^{n\times n}$, $i, j \in S$ such that*

$$\begin{bmatrix} \Phi_i & P_i G_i \\ \star & -\rho^2 I \end{bmatrix} < 0 \tag{4}$$

$$c_2 = \frac{e^{\alpha T}[c_1 \lambda_{max}(\widehat{P}_i) + \rho^2 \tau]}{\lambda_{min}(\widehat{P}_i)}, \quad \widehat{P}_i = M_i^{-\frac{1}{2}} P_i M_i^{-\frac{1}{2}} \tag{5}$$

where

$$\Phi_i = A_i^T P_i + P_i A_i + \sum_{j=1}^{N} \pi_{ij} P_j + \sum_{j=1, j\neq i}^{N} \left\{\frac{\lambda_{ij}^2}{4} V_{ij} + (P_j - P_i) V_{ij}^{-1} (P_j - P_i)\right\} - \alpha P_i$$

Proof Let $C[0, T]$ be the space of continuous functions. Then $\{(x(t), r(t)), t \geq 0\}$ is a semi-Markovian process with initial state (x_0, r_0). We choose stochastic Lyapunov–Krasovskii functional as

$$V(x(t), i, t) = x^T(t) P_i x(t) \tag{6}$$

where $P_i > 0, r(t) = i \in S$.

The Markovian switching system and semi-Markovian switching system are governed by different stochastic processes, so the infinitesimal generator of the Lyapunov–Krasovskii functional for the semi-Markovian switching system is different from the one for general Markovian switching system. According to the definition of infinitesimal operator Γ of the Lyapunov–Krasovskii functional, we have the following equality

$$\Gamma V(x(t), i, t) = \lim_{\Delta t \to 0} \frac{\mathscr{E}\{V(x(t+\Delta t), r(t+\Delta t), t+\Delta t)|x(t), r(t) = i, t\} - V(x(t), i, t)}{\Delta t} \tag{7}$$

where Δt is a small positive number. Inspired by [28], we obtain the infinitesimal generator of the Lyapunov–Krasovskii functional along the trajectory of semi-Markovian switching system (1) with $u(t) = 0$.

$$\begin{aligned}\Gamma V(x(t), i, t) &= x^T(t)[\sum_{j=1}^{N} \pi_{ij}(h) P_j] x(t) \\ &\quad + 2x^T(t) P_i [A_i x(t) + G_i \delta(t)] \\ &= x^T(t)[A_i^T P_i + P_i A_i + \sum_{j=1}^{N} \pi_{ij}(h) P_j] x(t) \\ &\quad + x^T(t) P_i G_i \delta(t) + \delta^T(t) G_i^T P_i x(t)\end{aligned} \tag{8}$$

Considering $\pi_{ij}(h) = \pi_{ij} + \Delta\pi_{ij}$, $\Delta\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \Delta\pi_{ij}$ and employing Lemma 2.3, we have

$$\begin{aligned}\sum_{j=1}^{N} \pi_{ij}(h) P_j &= \sum_{j=1}^{N} \pi_{ij} P_j + \sum_{j=1, j\neq i}^{N} \Delta\pi_{ij} P_j + \Delta\pi_{ii} P_i \\ &= \sum_{j=1}^{N} \pi_{ij} P_j + \sum_{j=1, j\neq i}^{N} \Delta\pi_{ij}(P_j - P_i) \\ &= \sum_{j=1}^{N} \pi_{ij} P_j + \sum_{j=1, j\neq i}^{N} \left[\frac{1}{2}\Delta\pi_{ij}(P_j - P_i) \right. \\ &\quad \left. + \frac{1}{2}\Delta\pi_{ij}(P_j - P_i)\right] \\ &\leq \sum_{j=1}^{N} \pi_{ij} P_j + \sum_{j=1, j\neq i}^{N} \left[\frac{\lambda_{ij}^2}{4} V_{ij} \right. \\ &\quad \left. + (P_j - P_i) V_{ij}^{-1} (P_j - P_i)\right]\end{aligned}$$

So we can rewrite (8) into the following form

$$\begin{aligned}\Gamma V(x(t), i, t) &\leq x^T(t) \left[A_i^T P_i + P_i A_i + \sum_{j=1}^{N} \pi_{ij} P_j \right. \\ &\quad \left. + \sum_{j=1, j\neq i}^{N} \left\{\frac{\lambda_{ij}^2}{4} V_{ij} + (P_j - P_i) V_{ij}^{-1} (P_j - P_i)\right\}\right] x(t) \\ &\quad + x^T(t) P_i G_i \delta(t) + \delta^T(t) G_i^T P_i x(t) \\ &= \begin{bmatrix} x^T(t) & \delta^T(t) \end{bmatrix} \begin{bmatrix} \Phi_i & P_i G_i \\ \star & -\rho^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} \\ &\quad + \alpha x^T(t) P_i x(t) + \rho^2 \delta^T(t) \delta(t)\end{aligned} \tag{9}$$

From (4), we obtain the following inequality

$$\Gamma V(x(t), i, t) \leq \alpha V(x(t), i, t) + \rho^2 \delta^T(t) \delta(t)$$

Multiplying the above inequality by $e^{-\alpha t}$, we have

$$\Gamma[e^{-\alpha t} V(x(t), i, t)] \leq \rho^2 e^{-\alpha t} \delta^T(t) \delta(t)$$

By Lemma 2.1, we can obtain

$$\begin{aligned}& V(x(t), i, t) - e^{\alpha t} V(x_0, i, t_0) \\ &\quad \leq \rho^2 e^{\alpha t} \int_0^t e^{-\alpha s} \delta^T(s) \delta(s) \mathrm{d}s\end{aligned}$$

According to Assumption 2.1, then we have

$$\begin{aligned}V(x(t), i, t) &\leq e^{\alpha t} \left[V(x_0, i, t_0) \right. \\ &\quad \left. + \rho^2 \int_0^t e^{-\alpha s} \delta^T(s) \delta(s) \mathrm{d}s\right] \\ &\leq e^{\alpha t} \left[V(x_0, i, t_0) + \rho^2 \int_0^t \delta^T(s) \delta(s) \mathrm{d}s\right] \\ &\leq e^{\alpha t} [V(x_0, i, t_0) + \rho^2 \tau]\end{aligned}$$

Define $\widehat{P_i} = M_i^{-\frac{1}{2}} P_i M_i^{-\frac{1}{2}}$ and utilize Lemma 2.2, we know that

$$V(x(t), i, t) \le e^{\alpha t}[c_1 \lambda_{max}(\widehat{P_i}) + \rho^2 \tau], \quad \forall t \in [0, T] \tag{10}$$

$$\begin{aligned} V(x(t), i, t) &= x^T(t) P_i x(t) \\ &\ge \lambda_{min}(\widehat{P_i}) x^T(t) M_i x(t), \quad \forall t \in [0, T] \end{aligned} \tag{11}$$

Combining with (10) and (11), we have

$$\mathscr{E}\{x^T(t) M_i x(t)\} \le \frac{e^{\alpha t}[c_1 \lambda_{max}(\widehat{P_i}) + \rho^2 \tau]}{\lambda_{min}(\widehat{P_i})} < c_2$$

Hence, by Definition 2.1, semi-Markovian switching system (1) can be finite-time bounded with respect to (c_1, c_2, T, M_i, τ), which completes this proof. □

Remark 3.1 The finite-time boundedness problem for semi-Markovian switching systems is considered for the first time in Theorem 3.1. The finite-time bounded condition is in the form of matrix inequalities but not the linear matrix inequalities, because Φ_i has nonlinear term $\sum_{j=1, j\neq i}^{N} (P_j - P_i) V_{ij}^{-1} (P_j - P_i)$ in (4).

Remark 3.2 It should be noticed that almost all the existing results on semi-Markovian switching systems presume that each mode possesses a single distribution of sojourn-time with certain parameters in a semi-Markov chain. By designing the Lyapunov–Krasovskii functional and employing the Lyapunov theory to establish criteria of stability analysis or control synthesis for semi-Markovian switching systems, the new challenge and the inherent difficulties mainly lie in how the probability density function information of the sojourn-time can be completely used in deriving the infinitesimal generator Γ of the Lyapunov–Krasovskii functional $V(x(t), r(t), t)$.

3.2 Finite-time H_∞ state feedback stabilization

Theorem 3.2 *Under the circumstance of $u(t) = 0$, given $c_1 > 0$, $T > 0$, $M_i > 0, i \in S$, $\tau > 0$, semi-Markovian switching system (1) is said to be H_∞ finite-time bounded with respect to (c_1, c_2, T, M_i, τ), if there exist positive constants $\alpha > 0$, $\rho > 0$, symmetric positive definite matrices $P_i \in \mathbb{R}^{n\times n}$, $i \in S$, symmetric positive definite matrices $V_{ij} \in \mathbb{R}^{n\times n}$, $i, j \in S$ such that*

$$\begin{bmatrix} \Phi_i + C_i^T C_i & P_i G_i + C_i^T H_i \\ \star & -\rho^2 I + H_i^T H_i \end{bmatrix} < 0 \tag{12}$$

where Φ_i, c_2 and $\widehat{P_i}$ are identical to those in Theorem 3.1.

Proof We choose the same stochastic Lyapunov–Krasovskii functional as (6), then we can obtain the following equality on the basis of proof procedure.

$$\begin{aligned} \Gamma V(x(t), i, t) &\le \begin{bmatrix} x^T(t) & \delta^T(t) \end{bmatrix} \begin{bmatrix} \Phi_i & P_i G_i \\ \star & -\rho^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} \\ &\quad + \alpha x^T(t) P_i x(t) + \rho^2 \delta^T(t) \delta(t) \\ &= \begin{bmatrix} x^T(t) & \delta^T(t) \end{bmatrix} \begin{bmatrix} \Phi_i & P_i G_i \\ \star & -\rho^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} \\ &\quad + \alpha x^T(t) P_i x(t) + \rho^2 \delta^T(t) \delta(t) \\ &= \begin{bmatrix} x^T(t) & \delta^T(t) \end{bmatrix} \begin{bmatrix} \Phi_i & P_i G_i \\ \star & -\rho^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} \\ &\quad + y^T(t) y(t) - y^T(t) y(t) + \alpha x^T(t) P_i x(t) \\ &\quad + \rho^2 \delta^T(t) \delta(t) \\ &= \begin{bmatrix} x^T(t) & \delta^T(t) \end{bmatrix} \begin{bmatrix} \Phi_i & P_i G_i \\ \star & -\rho^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} x^T(t) & \delta^T(t) \end{bmatrix} \begin{bmatrix} C_i^T C_i & C_i^T H_i \\ \star & H_i^T H_i \end{bmatrix} \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} \\ &\quad + \alpha x^T(t) P_i x(t) + \rho^2 \delta^T(t) \delta(t) - y^T(t) y(t) \\ &= \begin{bmatrix} x^T(t) & \delta^T(t) \end{bmatrix} \begin{bmatrix} \Phi_i + C_i^T C_i & P_i G_i + C_i^T H_i \\ \star & -\rho^2 I + H_i^T H_i \end{bmatrix} \\ &\quad \begin{bmatrix} x(t) \\ \delta(t) \end{bmatrix} + \alpha x^T(t) P_i x(t) + \rho^2 \delta^T(t) \delta(t) - y^T(t) y(t) \end{aligned} \tag{13}$$

According to (12) and (13), we can obtain

$$\begin{aligned} \Gamma V(x(t), i, t) &\le \alpha V(x(t), i, t) + \alpha x^T(t) P_i x(t) \\ &\quad + \rho^2 \delta^T(t) \delta(t) - y^T(t) y(t) \end{aligned}$$

i.e.

$$\Gamma V(x(t), i, t) \le e^{-\alpha t}[\rho^2 \delta^T(t) \delta(t) - y^T(t) y(t)] \tag{14}$$

Under zero initial condition $x_{t_0} = 0$, $t_0 = 0$, by Lemma 2.1 we have

$$e^{-\alpha t} V(x(t), i, t) \le \int_0^t e^{-\alpha s}[\rho^2 \delta^T(s) \delta(s) - y^T(s) y(s)] ds \tag{15}$$

It is noted that $\delta(t)$ is an arbitrary external disturbance and it has no relation with semi-Markovian process $r(t)$. (15) implies that

$$\mathscr{E}\left\{\int_0^t e^{-\alpha s} y^T(s)y(s)\mathrm{d}s\right\} \le \int_0^t e^{-\alpha s}\rho^2\delta^T(s)\delta(s)\mathrm{d}s$$

Then for all $t \in [0, T]$, we have

$$\begin{aligned} e^{-\alpha T}\mathscr{E}\left\{\int_0^t y^T(s)y(s)\mathrm{d}s\right\} &\le \mathscr{E}\left\{\int_0^t e^{-\alpha s} y^T(s)y(s)\mathrm{d}s\right\} \\ \le \int_0^t e^{-\alpha s}\rho^2\delta^T(s)\delta(s)\mathrm{d}s &\le \rho^2\int_0^T \delta^T(s)\delta(s)\mathrm{d}s \end{aligned} \tag{16}$$

Hence, it is held that

$$\mathscr{E}\left\{\int_0^T y^T(s)y(s)\mathrm{d}s\right\} \le e^{\alpha T}\rho^2\int_0^T \delta^T(s)\delta(s)\mathrm{d}s$$

where $\gamma = \sqrt{e^{\alpha T}}\rho$ in the form of Definition 2.2.

Therefore, by Definition 2.2, semi-Markovian switching system (1) can be H_∞ finite-time bounded with respect to (c_1, c_2, T, M_i, τ), which completes this proof. □

Theorem 3.3 *Given $c_1 > 0$, $T > 0$, $M_i > 0, i \in S$, $\tau > 0$, semi-Markovian switching system (1) with state feedback controller (2) is finite-time H_∞ state feedback stabilizable with respect to (c_1, c_2, T, M_i, τ), if there exist positive constants $\alpha > 0$, $\rho > 0$, symmetric positive definite matrices $X_i \in \mathbb{R}^{n\times n}$, $i \in S$, symmetric positive definite matrices $V_{ij}, W_{ij}, i, j \in S, i \ne j$, matrices $J_{ij}, i, j \in S, i \ne j$ and matrices Y_i, $i \in S$ with appropriate dimensions such that*

$$\begin{bmatrix} \Omega_i & G_i & X_iC_i^T + Y_i^T D_i^T & \mathcal{J}_i & \mathcal{X}_i \\ \star & -\rho^2 I & H_i^T & 0 & 0 \\ \star & \star & -I & 0 & 0 \\ \star & \star & \star & -\mathcal{V}_i & 0 \\ \star & \star & \star & \star & -\mathcal{W}_i \end{bmatrix} < 0 \tag{17}$$

$$\begin{aligned} c_2 &= e^{\alpha T}\left[c_1\frac{\lambda_{max}(\widehat{X}_i)}{\lambda_{min}(\widehat{X}_i)} + \rho^2\tau\lambda_{max}(\widehat{X}_i)\right], \\ \widehat{X}_i &= M_i^{\frac{1}{2}} X_i M_i^{\frac{1}{2}} \end{aligned} \tag{18}$$

where

$$\begin{aligned} \Omega_i &= X_iA_i^T + A_iX_i + Y_i^T B_i^T + B_iY_i + (\pi_{ii} - \alpha)X_i \\ \mathcal{J}_i &= [J_{i1}, J_{i2}\ldots, J_{i(i-1)}\ \ J_{i(i+1)}\ldots, J_{iN}] \\ \mathcal{V}_i &= diag\{V_{i1}, V_{i2}, \ldots, V_{i(i-1)}, V_{i(i+1)}, \ldots, V_{iN}\} \\ \mathcal{W}_i &= diag\{W_{i1}, W_{i2}, \ldots, W_{i(i-1)}, W_{i(i+1)}, \ldots, W_{iN}\} \\ \mathcal{X}_i &= [\underbrace{X_iX_i\ldots X_iX_i}_{N-1}] \end{aligned}$$

other notations are identical to those in Theorem 3.1.

Moreover, the required H_∞ feedback controller gains in the form of (2) are given by $K_i = Y_iX_i^{-1}$, $\forall i \in S$.

Proof Based on the result of Theorem 3.2, the closed-loop system (3) is investigated and it can be easily obtained

$$\begin{bmatrix} \widetilde{\Phi}_i & P_iG_i + (C_i + D_iK_i)^T H_i \\ \star & -\rho^2 I + H_i^T H_i \end{bmatrix} < 0 \tag{19}$$

where $\widetilde{\Phi}_i = \Phi_i + K_i^T B_i^T P_i + P_iB_iK_i + (C_i + D_iK_i)^T(C_i + D_iK_i)$.

By Schur complement lemma, which implies that

$$\begin{bmatrix} \Phi_i + K_i^T B_i^T P_i + P_iB_iK_i & P_iG_i & (C_i + D_iK_i)^T \\ \star & -\rho^2 I & H_i^T \\ \star & \star & -I \end{bmatrix} < 0 \tag{20}$$

Due to the existence of the nonlinear terms $(P_j - P_i)V_{ij}^{-1}(P_j - P_i)$, $K_i^T B_i^T P_i$ and $P_iB_iK_i$, the matrix equality is difficult to be solved. In order to obtain the desired controller K_i, we will decompose it into linear matrix inequality. Let us define

$$\Psi_i = A_i^T P_i + P_iA_i + K_i^T B_i^T P_i + P_iB_iK_i + \sum_{j=1, j\ne i}^{N} R_{ij} + (\pi_{ii} - \alpha)P_i$$

where $R_{ij} = \frac{\lambda_{ij}^2}{4}V_{ij} + \pi_{ij}P_j$, $j \ne i$.

Then (20) can be rewritten into the following form

$$\begin{bmatrix} \Psi_i & P_iG_i & (C_i + D_iK_i)^T \\ \star & -\rho^2 I & H_i^T \\ \star & \star & -I \end{bmatrix} + \mathrm{diag}\left\{\sum_{j=1, j\ne i}^{N} (P_j - P_i)V_{ij}^{-1}(P_j - P_i), 0, 0\right\} < 0$$

By Schur complement lemma, which implies that

$$\begin{bmatrix} \Psi_i & P_iG_i & (C_i+D_iK_i)^T & \zeta_i \\ \star & -\rho^2 I & H_i^T & 0 \\ \star & \star & -I & 0 \\ \star & \star & \star & -\mathcal{V}_i \end{bmatrix} < 0 \tag{21}$$

where

$$\zeta_i = [P_i - P_1 \quad \ldots \quad P_i - P_{i-1} \quad P_i - P_{i+1} \quad P_i - P_N]$$

Pre- and post- multiplying the inequality (21) by $diag\{P_i^{-1}, I, I, I\}$ and letting

$$X_i = P_i^{-1},\ Y_i = K_iX_i,\ J_{ij} = I - X_iX_j^{-1},\quad j \neq i,$$
$$W_{ij} = R_{ij}^{-1},\quad j \neq i$$

we obtain

$$\begin{bmatrix} \Omega_i & G_i & X_iC_i^T + Y_i^TD_i^T & \mathcal{J}_i \\ \star & -\rho^2 I & H_i^T & 0 \\ \star & \star & -I & 0 \\ \star & \star & \star & -\mathcal{V}_i \end{bmatrix} + \text{diag}\left\{\sum_{j=1, j\neq i}^{N} X_iR_{ij}X_i, 0, 0, 0\right\} < 0$$

By Schur complement lemma again, we know (17) is held. On the other hand, noticing $\widehat{P}_i = M_i^{-\frac{1}{2}}P_iM_i^{-\frac{1}{2}}$, we easily obtain

$$\widehat{X}_i \triangleq \widehat{P}_i^{-1} = M_i^{\frac{1}{2}}X_iM_i^{\frac{1}{2}}$$

and we know $\lambda_{\max}(\widehat{X}_i) = \frac{1}{\lambda_{\min}(\widehat{P}_i)}$. Then (18) is equivalent to (5).

Therefore, according to Definition 2.3, semi-Markovian switching system (1) with state feedback controller (2) is finite-time H_∞ state feedback stabilized by the state feedback controller (2) with $K_i = Y_iX_i^{-1},\ \ \forall i \in S$, which completes this proof.

Remark 3.3 The finite-time H_∞ stabilization condition for semi-Markovian switching system is proposed in Theorem 3.3, and the finite-time H_∞ state feedback controller has been solved in the form of linear matrix inequalities (LMIs), which can be solved by utilizing the LMI toolbox in Matlab.

Remark 3.4 With the assumption that there is a priori information of the upper and lower bounds of the probability density function, it is worth mentioning that one can easily extend the main results to the case under input saturation [49,50] by the techniques of linear matrix inequalities.

4 Numerical examples

In this section, we will utilize a semi-Markovian switching model over cognitive radio networks [51,52] to illustrate the usefulness and effectiveness of the approaches presented in the above sections.

Example 4.1 In [53], it indicates that cognitive radio systems hold promise in the design of large-scale systems due to huge needs of bandwidth during interaction and communication between subsystems. A semi-Markov process has been used to represent the switch between different states because the channel stays in each state are independent and identically distributed random variables following certain probability distribution functions. However, in this paper we consider each channel has four states and the switch between the modes is governed by a semi-Markov process taking values in $S = \{1, 2, 3, 4\}$. Furthermore, in the cognitive radio structure, it is assumed that at each step the sensor in cognitive radio infrastructure scans only one channel. This assumption avoids the use of costly and complicated multichannel sensors.

In this example, we assume that there are two channels to be sensed and each channel has four modes over cognitive radio links. Moreover, each channel is characterized by a Weibull semi-Markov process. So we can obtain a semi-Markovian switching system which can be described by (1). The system parameters are described as follows:

Mode 1

$$A_1 = \begin{bmatrix} -2 & -12 \\ -1 & 1 \end{bmatrix},\quad B_1 = \begin{bmatrix} 8 \\ 9 \end{bmatrix},\quad G_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$C_1 = [5, 7],\quad D_1 = 0.2,\quad H_1 = 0.8$$

Mode 2

$$A_2 = \begin{bmatrix} -6 & 5 \\ 6 & 4 \end{bmatrix},\quad B_2 = \begin{bmatrix} 4 \\ 7 \end{bmatrix},\quad G_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$C_2 = [1, 10],\quad D_2 = 0.3,\quad H_2 = 0.9$$

Mode 3

$$A_3 = \begin{bmatrix} 6 & 2 \\ 9 & -6 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 9 \\ 2 \end{bmatrix}, \quad G_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$C_3 = [9, 5], \quad D_3 = 0.2, \quad H_3 = 0.6$$

Mode 4

$$A_4 = \begin{bmatrix} 2 & -5 \\ 0 & 9 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad G_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$C_4 = [7, 1], \quad D_4 = 0.7, \quad H_4 = 0.1$$

The transition rates of semi-Markovian switching system in this model are given as follows
For mode 1

$$\pi_{11}(h) \in (-1.30, -1.10), \quad \pi_{12}(h) \in (0.20, 0.40)$$
$$\pi_{13}(h) \in (0.40, 0.60), \quad \pi_{14}(h) \in (0.30, 0.50)$$

For mode 2

$$\pi_{21}(h) \in (0.10, 0.30), \quad \pi_{22}(h) \in (-1.20, -0.80)$$
$$\pi_{23}(h) \in (0.25, 0.35), \quad \pi_{24}(h) \in (0.42, 0.58)$$

For mode 3
$$\pi_{31}(h) \in (0.74, 0.86), \quad \pi_{32}(h) \in (0.15, 0.25)$$
$$\pi_{33}(h) \in (-1.40, -1.20), \quad \pi_{34}(h) \in (0.23, 0.37)$$

For mode 4
$$\pi_{41}(h) \in (0.27, 0.33), \quad \pi_{42}(h) \in (0.14, 0.26)$$
$$\pi_{43}(h) \in (0.48, 0.72), \quad \pi_{44}(h) \in (-1.28, -0.92)$$

Then we can obtain the parameters $\pi_{ij}, \lambda_{i,j}, i, j \in S = \{1, 2, 3, 4\}$.

$$\pi_{11} = -1.20, \ \pi_{12} = 0.30, \ \pi_{13} = 0.50, \ \pi_{14} = 0.40$$
$$\pi_{21} = 0.20, \ \pi_{22} = -1.0, \ \pi_{23} = 0.30, \ \pi_{14} = 0.50$$
$$\pi_{31} = 0.80, \ \pi_{32} = 0.20, \ \pi_{33} = -1.30, \ \pi_{34} = 0.30$$
$$\pi_{41} = 0.30, \ \pi_{42} = 0.20, \ \pi_{43} = 0.60, \ \pi_{44} = -1.10$$
$$\lambda_{11} = \lambda_{12} = \lambda_{13} = \lambda_{14} = 0.10$$
$$\lambda_{21} = 0.10, \ \lambda_{22} = 0.20, \ \lambda_{23} = 0.05, \ \lambda_{24} = 0.08$$
$$\lambda_{31} = 0.06, \ \lambda_{32} = 0.05, \ \lambda_{33} = 0.10, \ \lambda_{34} = 0.07$$
$$\lambda_{41} = 0.03, \ \lambda_{42} = 0.06, \ \lambda_{43} = 0.12, \ \lambda_{44} = 0.18$$

Solving the LMIs in Theorem 3.3, we obtain

$$X_1 = \begin{bmatrix} 0.1238 & 0.0705 \\ 0.0705 & 0.0497 \end{bmatrix}, \quad Y_1 = [-0.0586, -0.1032],$$
$$X_2 = \begin{bmatrix} 0.2455 & -0.0227 \\ -0.0227 & 0.0949 \end{bmatrix}, \quad Y_2 = [-0.0526, -0.2859],$$
$$X_3 = \begin{bmatrix} 0.1240 & -0.0585 \\ -0.0585 & 0.2008 \end{bmatrix}, \quad Y_3 = [-0.2264, -0.1609],$$
$$X_4 = \begin{bmatrix} 0.1240 & -0.0585 \\ -0.0585 & 0.2008 \end{bmatrix}, \quad Y_4 = [-0.3330, -0.5743],$$

and $\rho = 2.2046$.

The finite-time H_∞ state feedback controller gains are given by

$$K_1 = [3.9520, -8.0262], \ K_2 = [-0.5260, -3.1627],$$
$$K_3 = [-2.5859, -1.5639], \ K_4 = [16.3720, -23.5048]$$

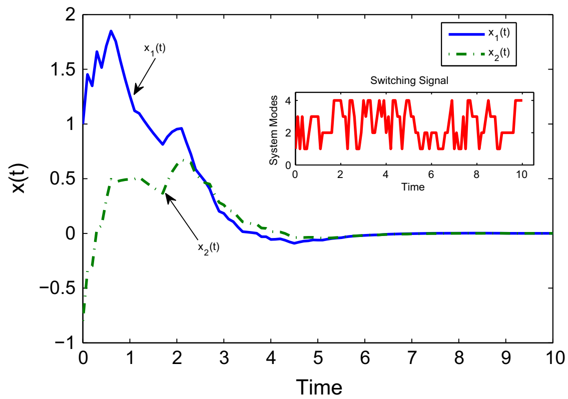

Fig. 1 State response of the closed-loop system with $\delta(t) = 0$

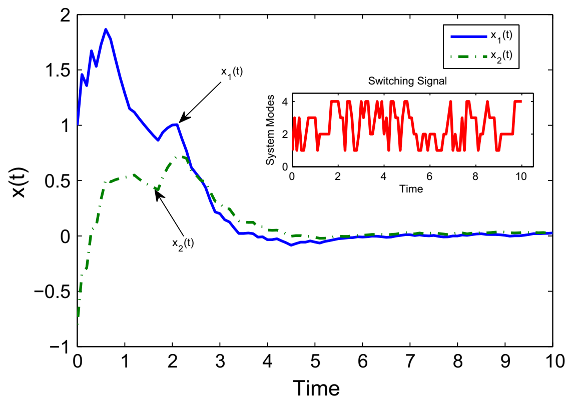

Fig. 2 State response of the closed-loop system with $\delta(t) \neq 0$

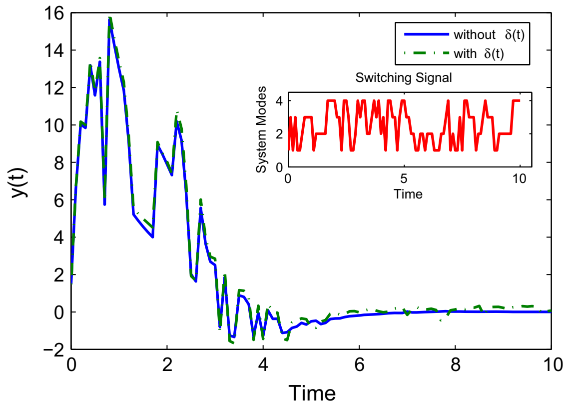

Fig. 3 Response of the output $y(t)$ corresponding to external disturbance

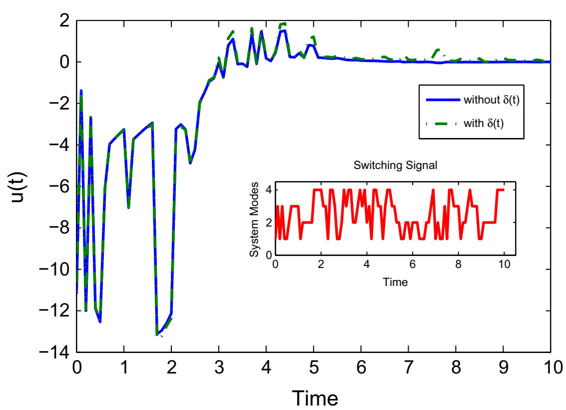

Fig. 4 Control input signal $u(t)$

Furthermore, trajectory figures of semi-Markovian switching system (1) with the given parameters are presented in Figs. 1, 2 and 3 to demonstrate the effectiveness of the design method. When $\delta(t) = 0$, the state response of the closed-loop system and the corresponding switching signal over the interval $[0, T]$ are described in Fig. 1. When $\delta(t) \neq 0$, the state response of the closed-loop system and the corresponding switching signal over the interval $[0, T]$ are shown in Fig. 2. Then, Fig. 3 shows the responses of the output $y(t)$ with $\delta(t) = 0$ and $\delta(t) \neq 0$ and Fig. 4 shows the control input signal $u(t)$ with $\delta(t) = 0$ and $\delta(t) \neq 0$. It can be seen that semi-Markovian switching system (1) is finite-time bounded and H_∞ finite-time bounded, which implies the given system can achieve finite-time H_∞ stabilization via the designed state feedback controller (2).

5 Conclusions

In this paper, the finite-time H_∞ stabilization problem has been initially investigated for a class of semi-Markovian switching systems. Based on the Lyapunov–Krasovskii functional, properties of semi-Markovian process and some matrix techniques, sufficient conditions have been established to ensure the finite-time boundedness, H_∞ finite-time boundedness and finite-time H_∞ state feedback stabilization for the given system, respectively. Meanwhile, a finite-time H_∞ state feedback controller has been given to guarantee finite-time H_∞ stabilization for the semi-Markovian switching system. Finally, a numerical example and simulations are provided to demonstrate the effectiveness and correctness of the above results. Future work will be focused on more complicated cases, such as nonlinear perturbations and uncertainties taken into consideration simultaneously.

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